This coursework will be marked anonymously
YOU MUST NOT PUT ANY INDICATION OF YOUR IDENTITY IN YOUR SUBMISSION

This coursework should take an average student who is up-to-date with tutorial work approximately 25 hours

Feedback and grades are normally made available within 15 working days of the coursework deadline

Learning Outcomes:

C. recognise the classic Black-Scholes model of asset price movement and understand risk-neutral pricing and the Black-Scholes formula and the use of implied volatility.
D. obtain lower and upper bounds for vanilla option prices, including American options, using analytic as well as financial methods.
E. apply a range of stochastic models to the movement of derivatives and their underlying asset price.
G. use Monte Carlo methods to price certain types of non-standard options.

Plagiarism is presenting somebody else's work as your own. It includes: copying information directly from the Web or books without referencing the material; submitting joint coursework as an individual effort; copying another student's coursework; stealing coursework from another student and submitting it as your own work. Suspected plagiarism will be investigated and if found to have occurred will be dealt with according to the procedures set down by the University. Please see your student handbook for further details of what is / isn't plagiarism.

All material copied or amended from any source (e.g. internet, books) must be referenced correctly according to the reference style you are using.

Your work will be submitted for plagiarism checking. Any attempt to bypass our plagiarism detection systems will be treated as a severe Assessment Offence.

Coursework Submission Requirements

- An electronic copy of your work for this coursework must be fully uploaded on the Deadline Date of **Wednesday 07/03/2018** using the link on the coursework Moodle page for MATH1046.
- For this coursework you must submit a single PDF document. In general, any text in the document must not be an image (i.e. must not be scanned) and would normally be generated from other documents (e.g. MS Office using "Save As .. PDF"). An exception to this is hand written mathematical notation, but when scanning do ensure the file size is not excessive.
- For this coursework you must also upload a single **ZIP** file containing supporting evidence.
- There are limits on the file size (see the relevant course Moodle page).
- Make sure that any files you upload are virus-free and not protected by a password or corrupted otherwise they will be treated as null submissions.
- Your work will not be printed in colour. Please ensure that any pages with colour are acceptable when printed in Black and White.
- You must NOT submit a paper copy of this coursework.
- All courseworks must be submitted as above. Under no circumstances can they be accepted by academic staff

The University website has details of the current Coursework Regulations, including details of penalties for late submission, procedures for Extenuating Circumstances, and penalties for Assessment Offences. See http://www2.gre.ac.uk/current-students/regs

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- **Detailed Specification**
  This coursework is to be completed individually.
- **Deliverables** Zip file containing report pdf file and Excel spreadsheets
- **Grading Criteria**
  Assessment Criteria
  See mark breakdown in specification.

MODELLING AND ANALYSIS OF FINANCIAL INSTRUMENTS

(Term 2, 2017/2018)

Coursework 1 of 1, 50% of course

IMPORTANT: you must use the price data (downloaded from http:\\finance.yahoo.com) for YOUR allocated share in answering all 4 questions. You may need to clean your data by removing holiday dates when the share was not traded and inserting dividends on the appropriate ex-dividend date. Presentation of your written solutions and your spreadsheet is important and you will lose marks for any careless or poorly organised work.

Use the assignment upload system on Moodle to upload a **single** zip file containing your report as a pdf and your Excel spreadsheet. All computed values should be clearly identified on your spreadsheet and summarised in your coursework report (but do not print out the detailed price data in your written report).

Enter data for your share in the following datasheet and include this as the first page in your written coursework submission.	
Datum	Value
Name of Your Share	
Share price in pence on the first trading day of Jan 2018	
Calculated Option Strike price (K) chosen for your share (see Q 2)	
Number of daily prices in your data for July-December 2017	
Your calculated result for the daily mean log-return (m) for July-December 2017	
Your calculated result for the daily volatility for July-December 2017.	

Dataset

You should download daily prices from either Yahoo or Google for your FTSE100 share. The time interval is first day of trading Jan 2016 – last day of trading Dec 2017 – i.e. two years data and approximately 504 data points. You should try to obtain dividend information for your share. The share price on the first day of trading of Jan 2018 will also be needed.

Question 1.

a) Sort and clean your data and then insert it into the coursework analysis spreadsheet. Make sure that the share prices are in the correct order (increasing time). Find and enter the dividends for your share within this period. Report if there have been any share splits. Select the last 6 months data from 2017 (July-Dec 2017) and calculate the mean (m) and standard deviation (s) of that six-month period's daily log return data. Work out the corresponding parameters of the log return model,

$$\ln\left(S_{t+\Delta t}/S_t\right) \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t, \sigma^2 \Delta t\right)$$

where $\Delta t = 1$ day =1/254 (approx.) years is the time interval between prices, σ is the annualised volatility and μ is the annual mean return. Use sampling theory to estimate the error in this volatility.

[10 marks]

b) Use **the entire 2 years data** to produce empirical graphs of the frequency distribution for 1-day, 5-day, 10-day and 20-day log returns and report on goodness of fit of the normal distribution. NB – when testing for normality remember that most tests require samples to be *independent*. Describe variance ratio tests for these returns and explain how these test the random walk hypothesis. Evaluate the RWH using these tests for your data.

[14 marks]

c) Use the estimates of μ, σ obtained in Q1 (a) and set S_0= share price on the first day of trading of Jan 2018. What is the probability distribution of the share price S_T at T = 3 months and T = 6 months, respectively? In what way are they different and why? What are the expected share prices for T = 3 months and T = 6 months?

[5 marks]

d) Using parameters as in Q1 (c), find the 95% prediction corridor (i.e. the sequence of 95% share price prediction intervals) of the share price at t = 2, 4, 6, 8, 12 months for 2018 and create a suitable plot of the results. Print this chart and include in your written coursework.

[5 marks]

Question 2.

Assume that the risk-free interest rate is 2.5% per year. Set up an Option strike price K at 110% x share price on the first day's trading of Jan 2018, rounded up to the nearest 10p. For example, if that closing price were 243p then 1.1 x 243 = 267.3 which results in K=270p, rounded up to the nearest 10p.

a) Determine the price on the **first day's trading of** Jan 2018 of a European Put option on your share, maturing in 6 months at the strike price K calculated above, using the Black-Scholes pricing formula. Use the annualised volatility for your share calculated in Q1 Part (a). Use a calculator, Statistical Tables and show your working. (You should check your answer with Excel but there are **no mark**s for reporting Excel calculations for this question or for part (b) below)

[5 marks]

b) Suppose your share issues annual dividends of 5%, payable in 3 months' time (end of March 2018). Determine the price on the **first day's trading of** Jan 2018 of a European Put option on your share, expiring in 6 months at the strike price K calculated above, **using the dividend-adjusted Black-Scholes formula**. Use a calculator, Statistical Tables and show working.

[5 marks]

c) Assume that you are a trader and have sold 10 Put options priced as in Q2 (a) above. How would you Delta hedge your exposure on this contract? How would your hedge be managed over the 6 months until the exercise date?

[8 marks]

d) Use the bounds on the price of an American Put Option given in the lecture notes and compute a price range for this American Option using the same contract data as above. How does the American price range compare to the European Put price?

[8 marks]

Question 3. (Uses the same data as that of Q2 for K, S_0, r and of Q1a for the volatility)

Set up a 2-step binomial model for your share for Jan-June 2018 using the following steps:

a) Identify the up and down factors **u** and **d** and the path probability **q** from your historic volatility for a 2-step binomial tree from t = 0 to t = T, where T = 6 months, using the formulae in the Binomial Tree Lecture Notes.

[4 marks]

b) Work out the branches of this 2-step binomial tree and calculate the possible share prices at T=6 months and then compute the expected share price at T=6 months. Use your **first day's trading of** Jan 2018 share price as the starting price. Compare this binomial tree expected share price with the expected price determined using log-normal probabilities in Q1(c) and comment on the comparison.

[7 marks]

c) Use risk-neutral probabilities (see Lecture Notes) to calculate the risk-neutral expected payoff of your Put Option and compute and compare the Binomial tree European Put Option price to the Black-Scholes European Put Option price in Q2(a).

[7 marks]

d) Show how dynamic hedging would be implemented at each stage of the tree, i.e. specify at each node, the combination of shares and cash required to hedge a **sold** put option, so that the payoff at T=6 months is matched exactly.

[7 marks]

e) Use the binomial tree to price an American Put using the same data as in part c). Compare the price with that of the European Put in Q3 c) and the bounds established in Q2d). Should the American Put be exercised early? Justify your decision.

[7 marks]

Question 4.

Use the Monte Carlo spreadsheet on Moodle and set up a Monte Carlo calculation of the price of the Put Option defined in Question 2. Comment quantitatively on the expected accuracy of the price, by analysing the distribution of repeated Put Option, price estimates. Compare the obtained price with the exact Black Scholes price. Plot the first five of the simulations together with share price corridor for the same 6 month period.

[8 marks]