L2411

FREQUENCY RESPONSE OF NETWORKS

1. Objectives

a) To investigate how capacitive and inductive reactance vary with frequency,
b) To plot the frequency response of a simple *RC* filter,
c) To plot the variation of current with frequency in a series *RLC* circuit and determine the resonant frequency and *Q* factor, and
d) To evaluate simple bandpass and bandstop filters.

Note on good experimental technique

In the set laboratory experiments that you undertake, the most important part is understanding the physical processes and the mathematical concepts involved. This is most likely to happen when you come across something that does not conform to theoretical prediction. Do not fret when this happens. Your experiment and your understanding of it, now really begins.

After eliminating any trivial causes that may have caused the problem, start to think about the experiment.

a) Is the equipment working within its normal bounds?
b) Are you operating the equipment correctly?
c) Start to question the validity of the experimental procedure.
d) Are there any reasons to question the validity of the model on which the theory was based?

Improve all these if necessary.

Your discussion in the report of the experiment should reflect this.

Hints for good experimental work

a) Come prepared for the laboratory exercise by being familiar with the theory and procedures involved.
b) Take running graphs; on the same graph with the experimental and the expected theoretical plot, if possible. Most calculators have some programming capability. Use this feature .You will then know immediately if something is going astray.

2. Theory

In the following theory, equations are stated without proof. It is left to the student to derive these expressions in the report.

1) Capacitive Reactance

If I_C and V_C are the r.m.s. values of current through, and voltage across a capacitor, then

$$X_C = \frac{V_C}{I_C} \qquad (1)$$

where X_C is known as the capacitive reactance and has units of ohms. If the capacitor has a value of C farads at a frequency of f Hz, then

$$X_C = \frac{1}{2\pi f C} \qquad (2)$$

2) Inductive Reactance

The ratio of voltage across an inductor to current through an inductor is known as the ***inductive reactance*** X_L, and

$$X_L = \frac{V_L}{I_L} \qquad (3)$$

where $X_L = 2\ \pi f L\ \ \Omega$.

3) Simple RC filter

A simple RC low-pass filter is shown in Fig. 1. The frequency response of this network is given by

$$G(j\omega) = \frac{V_0}{V_i} = \frac{1}{1 + j\omega CR} \qquad (4)$$

The magnitude response is given by

$$|G(j\omega)| = \frac{1}{\sqrt{1 + (\omega CR)^2}} \qquad (5)$$

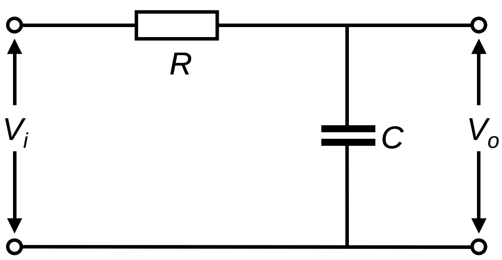

Fig. 1

and can be expressed in dBs as

$$20 \cdot \log_{10}|G(j\omega)| = -20 \cdot \log_{10}\sqrt{1 + (\omega CR)^2} \qquad (6)$$

When $\omega CR << 1$, the gain is $-20 \log_{10} 1 = 0$ dB
When $\omega CR = 1$, the gain is $-20 \log_{10} 2 = -3$ dB
When $\omega CR >> 1$, the gain is $-20 \log_{10}(\omega CR) = -20(\log_{10} CR + \log_{10}\omega)$

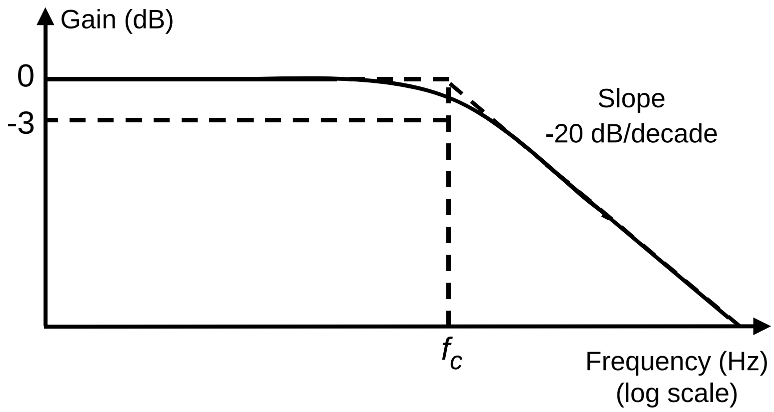

Fig. 2

It is evident that if ω is increased by a factor of 10 (1 decade) then the gain will reduce by 20 dB ***giving a slope of -20 dB/decade***. The frequency response is sketched in Fig. 2. The frequency $f_C = 1/(2\pi RC)$ is known as **the *cut-off frequency of the filter***.

4) Resonance

A series *RLC* circuit is shown in Fig. 3. Resonance occurs when the current *I* is in phase with the applied voltage *V*. This occurs when the inductive reactance and the capacitive reactance are equal. The frequency at which this occurs is called the ***resonant frequency*** and is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad (7)$$

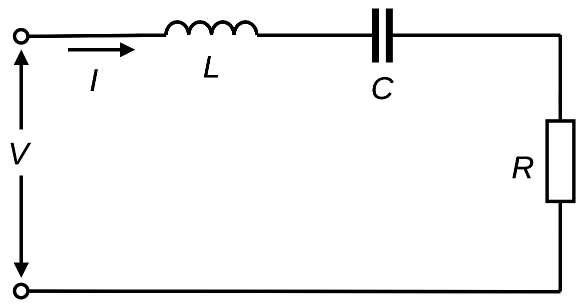

Fig. 3

The impedance of a series resonant circuit reaches a minimum value $Z = R$ at the resonant frequency, so that $I = V/R$. A typical sketch of the variation of current with frequency is show in Fig. 4.

The selectivity of a resonant circuit is expressed in terms of the ***quality*** or ***Q factor***. This can be determined from the circuit components or from the frequency response sketch.

$$Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad (8)$$

Also

$$Q = \frac{f_0}{B_w} \qquad (9)$$

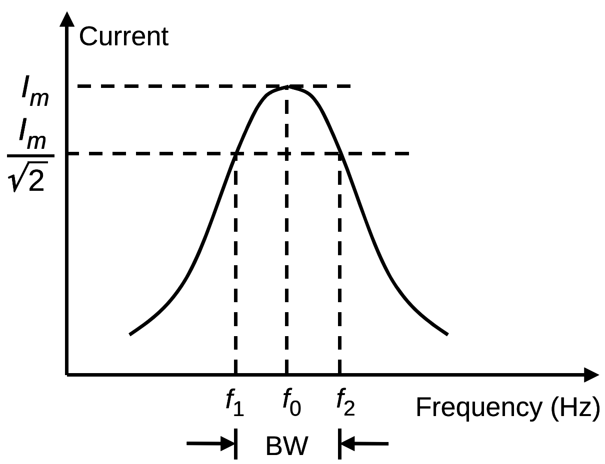

Fig. 4

where B_w is the ***bandwidth in Hz between the half-power frequencies*** f_2 and f_1, i.e.,

$$B_w = f_2 - f_1$$

3. Experimental Work

3.1 Apparatus

Patchboard and components
Farnell low-frequency (LF) Oscillator
Oscilloscope (CD1400 including 1 off CX1 449)
Digital or valve voltmeter
Digital frequency meter

3.2 Procedure

In all the following sections use a digital frequency meter to record the frequency of the LF oscillator.

1) Connect the LF oscillator to the series connection of a resistor and capacitor as shown in Fig. 5. Measure the voltage across the capacitor and the current through the capacitor at a number of frequencies between 1 kHz and 10 kHz. The current through the capacitor may be found by monitoring the voltage across the 100 Ω resistor. At one frequency use the CRO to record the current and voltage waveforms and sketch the result.

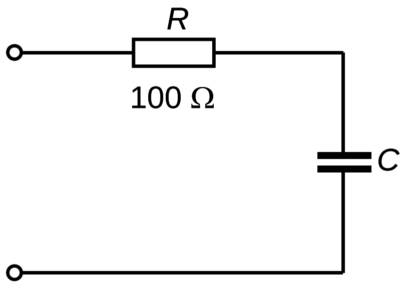

Fig. 5

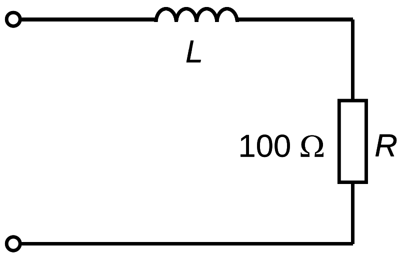

Fig. 6

2) Repeat the above section 1) for the series connection of a resistor and inductor as shown in Fig. 6. **Also measure the voltage across the inductor at 100 Hz. (Why do you need to do this measurement?)**

3) Measure the frequency response of the simple *RC* low-pass filter as shown in Fig. 1 over the frequency range 100 Hz to 100 kHz. Let R = 1 kΩ, C = 0.01 μF, and record the attenuation at 15 to 20 frequencies logarithmically spaced over this frequency range.

   Repeat the above procedure with R = 10 kΩ.

4) Connect the LF oscillator to the *RLC* series circuit shown in Fig. 3. With R = 100 Ω measure the voltage across the capacitor and inductor, and the current in the circuit at a number of frequencies between 1 kHz and 100 kHz. The current in the circuit may be found by measuring the voltage across the series resistor.

5) Bandpass the bandstop filters. The resonant circuit of experiment 4) can be used as the basis of a more effective filter.

(a) Construct the bandpass filter circuit of Fig. 7. Measure V_0 and V_i over the range of frequencies 1 kHz to 100 kHz.

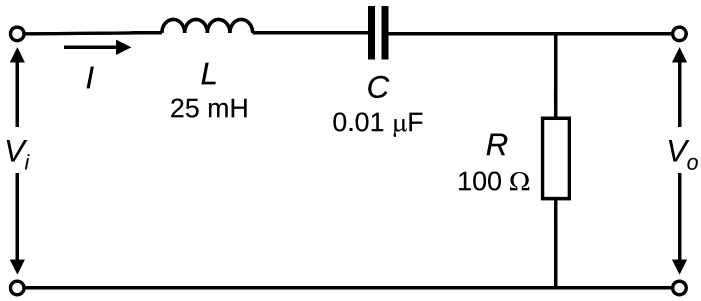

Fig. 7

(b) Construct the bandstop filter circuit of Fig. 8. Again, measure V_0 and V_i over the range of frequencies 1 kHz to 100 kHz.

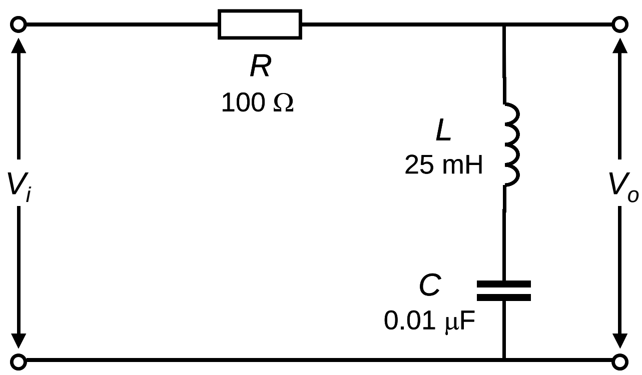

Fig. 8

(c) The Twin-T filter of Fig. 9 is usually more convenient to use as it does not require an inductor. Construct the filter with $R1 = R2 = 100\ \Omega$ and $C1 = C2 = 0.01\ \mu F$. Measure the input and output voltages over the frequency range 100 Hz to 10 kHz. Investigate the variation in performance of the filter if the resistor and/or capacitor values are changed.

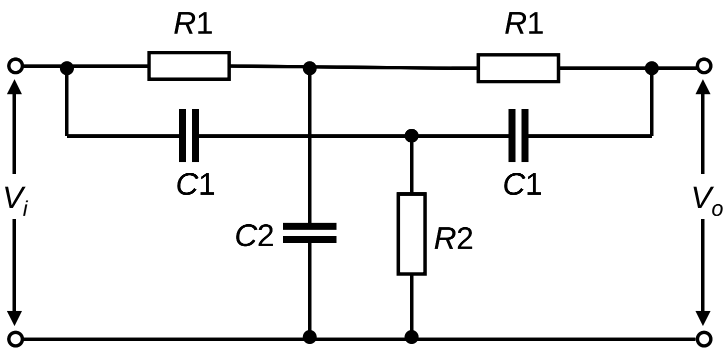

Fig. 9

For example: $R1 = R2 = 1000\ \Omega$, $C1 = C2 = 0.01\ \mu F$
$R1 = 1000\ \Omega$, $R2 = 100\ \Omega$, $C1 = C2 = 0.01\ \mu F$
$R1 = R2 = 1000\ \Omega$, $C1 = 0.01\ \mu F$, $C2 = 0.1\ \mu F$

3.3 Results

1) Plot a graph of $1/X_C$ against frequency and from this graph determine the value of capacitance.

2) Plot a graph of X_L against frequency and from this graph determine the value of inductance. Calculate the reactance that should be obtained at 100 Hz and compare this with the measured value.
3) Plot the two low-pass filter frequency responses as gain (in dB) against frequency. Use a suitable size of log-linear graph paper.

4) Plot V_L, V_C and I against frequency for the series RLC circuit. From the graph determine the resonant frequency and the Q of the tuned circuit. The values may be compared with the calculated values of f_0 and Q which are obtained by substituting the values for L, C and R in the expressions given in the theory.

5) Plot V_0/V_i against f over the appropriate range of frequencies for each of the bandpass and bandstop circuits.

4. Discussion of Results

Discuss all the results you have obtained **in relation to the theory** and in particular any discrepancies between theory and experiment, bearing in mind the errors and uncertainty in any experimental work. Your discussion should be based on an analysis of the experiment.

For the top mark, your report should include derivation of equations stated in the Theory section.

5. Conclusions

Your conclusions should include an evaluation of the measurement techniques used. Discuss the significance of results you have obtained.

6. References

[1] Hughes E., Electrical Technology, Longman Scientific: 7th ed., 1994, *or* Electrical and Electronic Technology, Prentice Hall: 8th ed., 2002, Chapter 21. (Library code TK 145 H8)

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