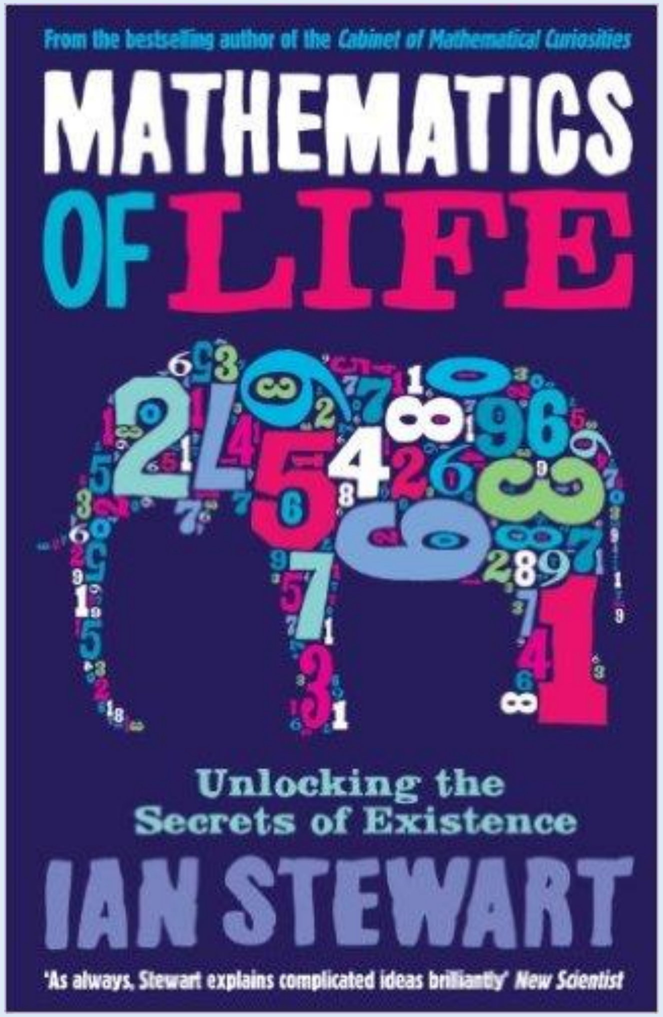
Life Sciences

Introduction

- Applications of mathematics in biology
- Source: Ian Stewart, *Mathematics of Life: Unlocking the Secrets of Existence* (Profile, 2011)



Mathematics in Biology

- Traditional mathematical modelling (eg differential equations) is an important tool in mathematical biology
- A large part of this course will investigate these applications
- But, particularly over the last 20 years or so, other kinds of mathematics have successfully found applications
- Including some you might not expect!

Peas and Genetics

- Gregor Mendel (1822-1884)
- A monk in Moravia (Czech Republic)
- In 1856 began experiments breeding peas
- Cross-bred pea plants with purple flowers and those with white flowers
- Looked at colour of flowers of the result

Two “white” plants

- All offspring white

Two “purple” plants

- Two possibilities
- All offspring purple
- ¾ of offspring purple and ¼ white

“White” and “purple” plants

- Two possibilities
- All offspring purple
- ½ of offspring purple and ½ white

The explanation

- Each plant contains two “factors” which determine the colour of the flowers
- These factors can be P (Purple flowers) or W (White flowers)
- If a plant has two W factors then the flowers are white
- If two P factors then flowers are purple
- If one P and one W then the flowers are always purple

The explanation

- When plants reproduce the offspring inherit one factor from each parent
- Which of the two is inherited is random

The explanation

- So for two purple parents:
- If they are both PP then offspring is PP – purple flowers
- If one is PP and one is PW then offspring is PP or PW – purple flowers

The explanation

- So for two purple parents:
- If both parents are PW then offspring is
  - PP with probability ¼
  - PW with probability ½
  - WW with probability ¼
- Giving ¾ purple flowers and ¼ white flowers

The explanation

- For two white parents:
- Both parents are WW and offspring must be WW - white

The explanation

- For one white and one purple parent:
- If WW and PP then offspring is WP – purple
- If WW and PW then half offspring are WW – white and half are WP - purple

Mendel’s conclusion

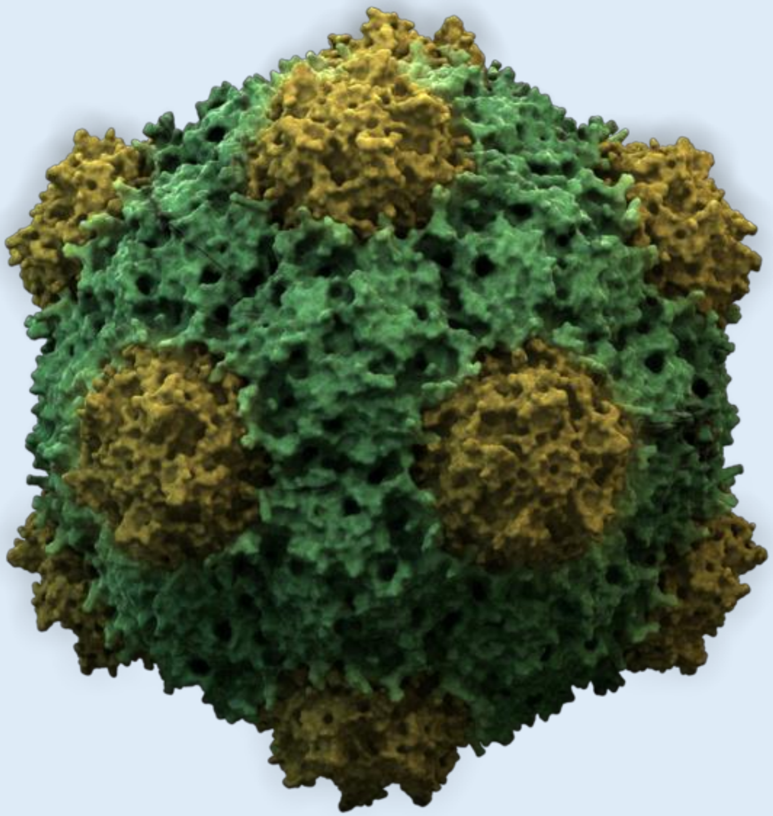
- Mendel deduced from the mathematics of combinations that there must be factors behaving in this way
- He published this in 1865: ignored and forgotten until 1890
- The mechanism (genes) not identified until second half of twentieth century

Viruses

- Viruses have two components – genes enclosed in a protein coat (capsid)
- Typically the coat is a regular shape which is as near spherical as possible
- Many are icosahedral – closest regular polyhedron to a sphere

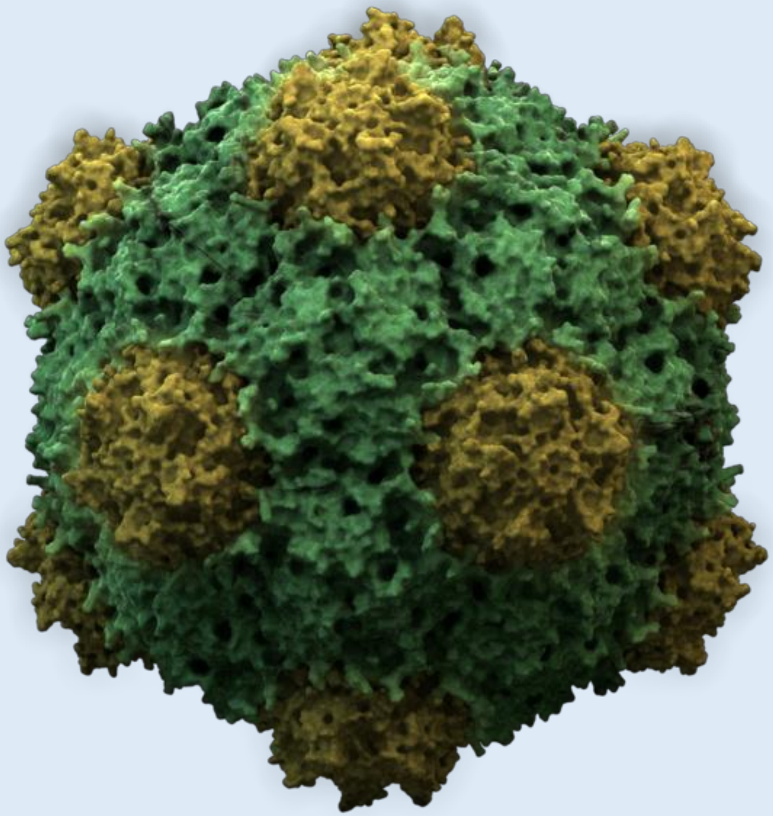
Cowpea Virus

- Looks icosahedral but not necessarily so at molecular level
- Graph theory imposes constraints
- $n - m + f = 2$ (Euler)
- Any polyhedron made up of hexagons and pentagons has exactly 12 pentagons



Cowpea Virus

- So graph theory helps us understand how viruses are structured
- Recent work has found that tiling in four or more dimensions gives more insights

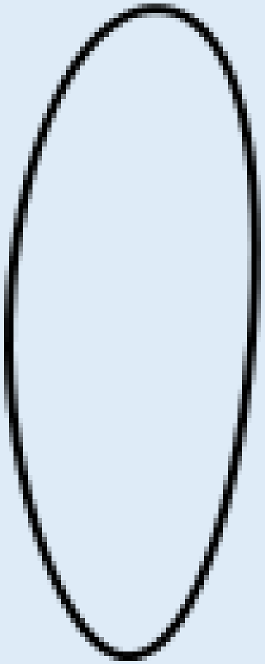


Disease modelling

- How does an epidemic behave?
- Depends on connections
- Graph theory – the maths of connections – helps us understand the spread of disease

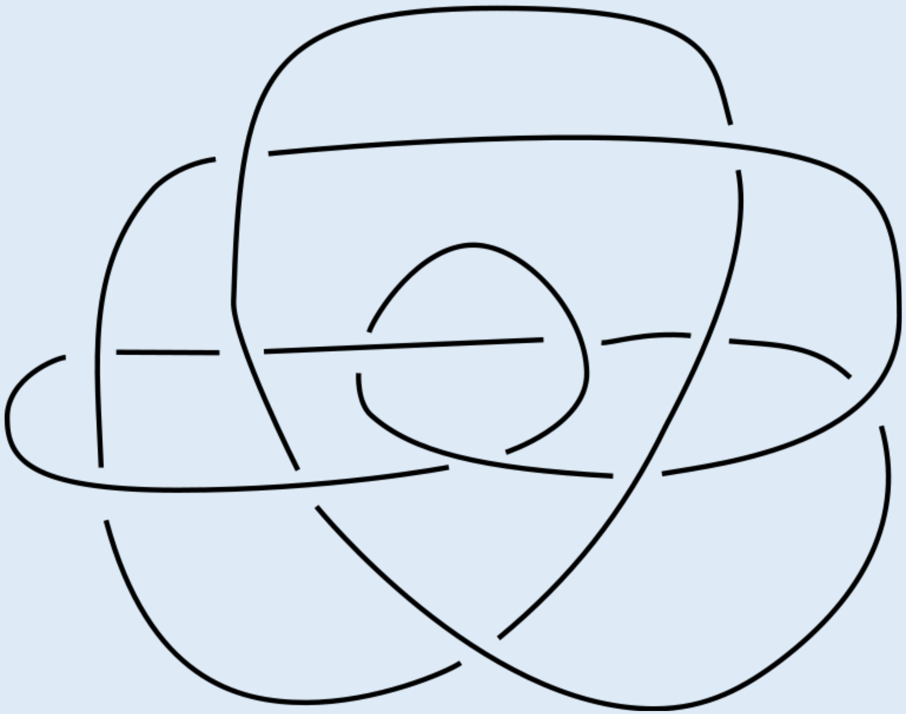
Knot theory

- The study of knots
- On the left, the “un-knot”
- On the right, a knot which is also the un-knot



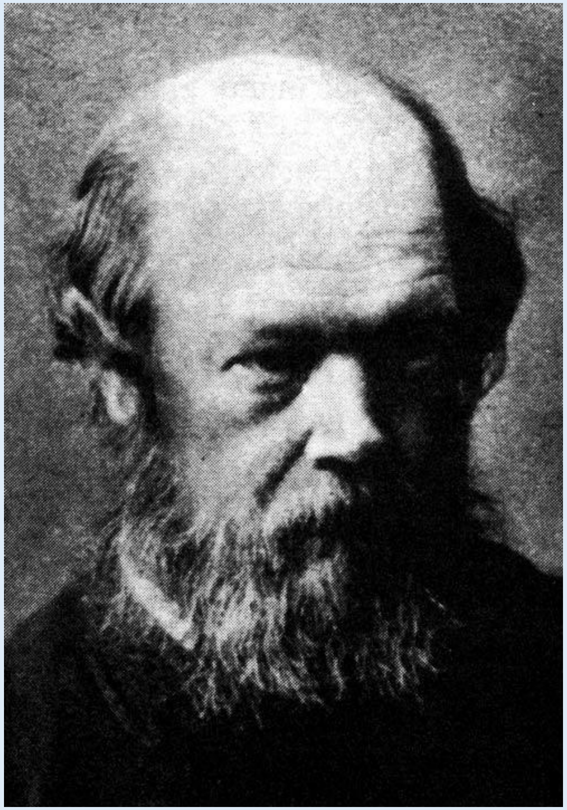
Knot theory

- Is this the unknot?
- Hard to tell!
- (It is, as it happens)
- Knot theory is difficult!



Knot theory

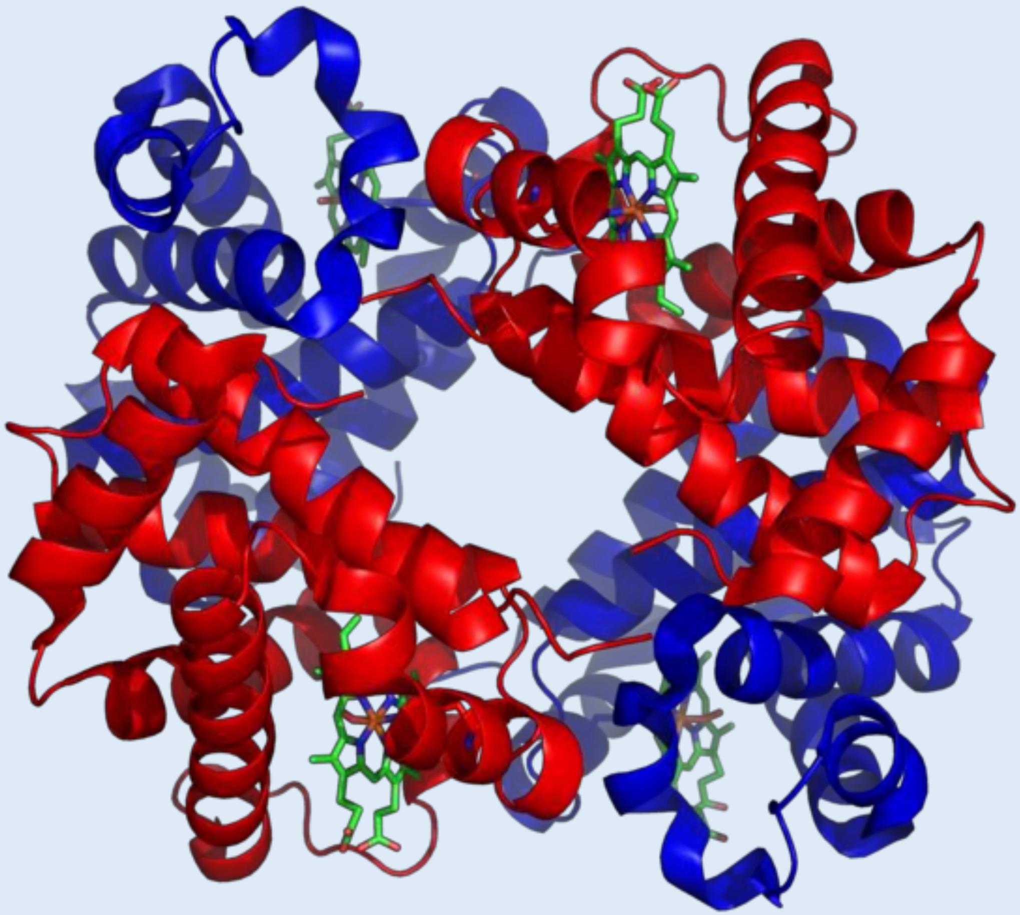
- Investigated by Peter Guthrie Tait, Edinburgh, second half of nineteenth century
- Trying to understand the structure of matter before atomic theory
- Then studied as a branch of topology in pure mathematics
- 1970s – William Thurston
- 1980s – Vaughan Jones and the Jones Polynomial



Knot theory in the 21st century

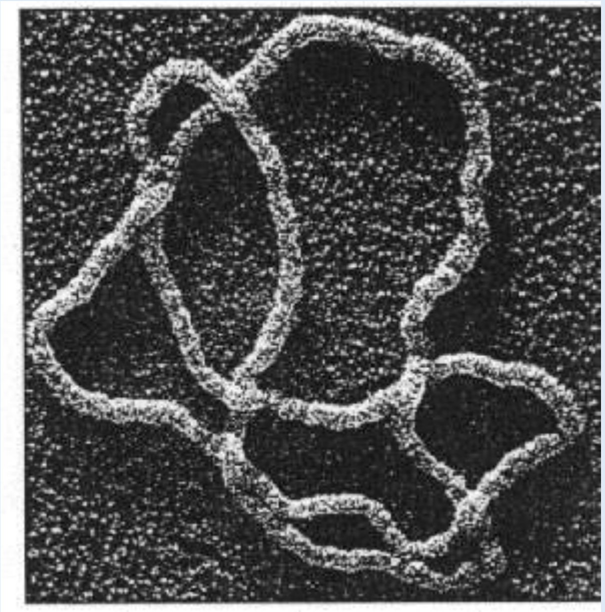
- Now fundamental to molecular biology
- How molecules are structured

Haemoglobin



Knot theory in the 21st century

- Used to understand the actions of enzymes in DNA
- Enzmes break and recombine strands of DNA
- Creating different kinds of knots



Spots and stripes

Spots and stripes

Spots and stripes

Spots and Stripes

- Alan Turing developed a mathematical theory of pattern formation which explains how patterns develop

The Paradox of the Plankton

- A general principle of evolutionary biology is that there is one species for any "niche" (Gause's Law)
- Because if two species were competing, one would "win" and drive the other out
- This does not happen with plankton, which are populations of many different species, in proportions which change over time, in the same habitat

The Paradox of the Plankton

- The explanation is that Gause’s Law assumes a steady-state equilibrium
- Chaos theory has shown us that not all systems end up in steady states
- In a non-equilibrium dynamics Gause’s Law does not hold, as the plankton show

Gait analysis

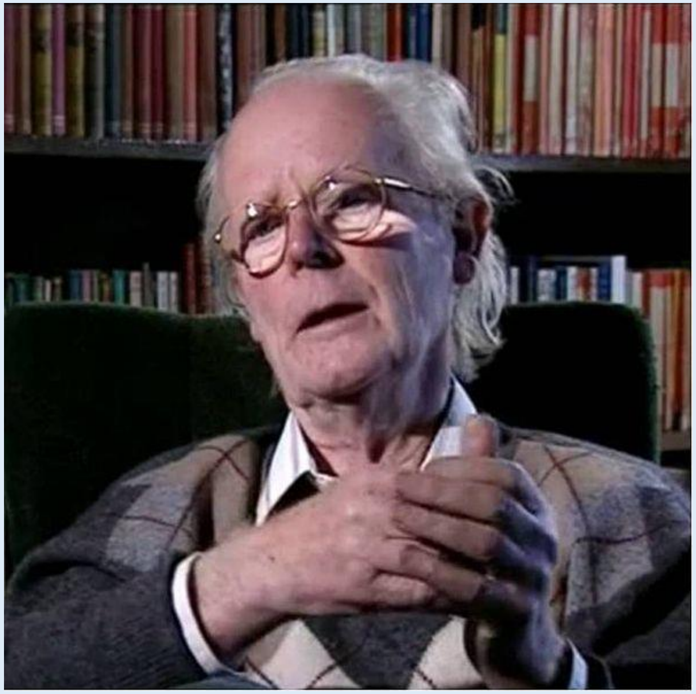
- Animals have characteristic gaits
- When a horse walks,
  - Legs move in turn
  - Each hits the ground in successive quarters of the cycle
  - Left back – left front – right back – right front
- In a trot, left front and right back legs hit the ground together, then right front and left back half a cycle later
- Two-legged gaits – walk, run, hop, skip and others

Gait analysis

- Network theory, applied to nerves and the brain, shows what gaits are possible
- Nice account in Ian Stewart's book of how, working with a colleague in Houston, they found that the mathematics showed an unusual and unlikely gait for horses,
- Which they recognised that evening in a rodeo show!

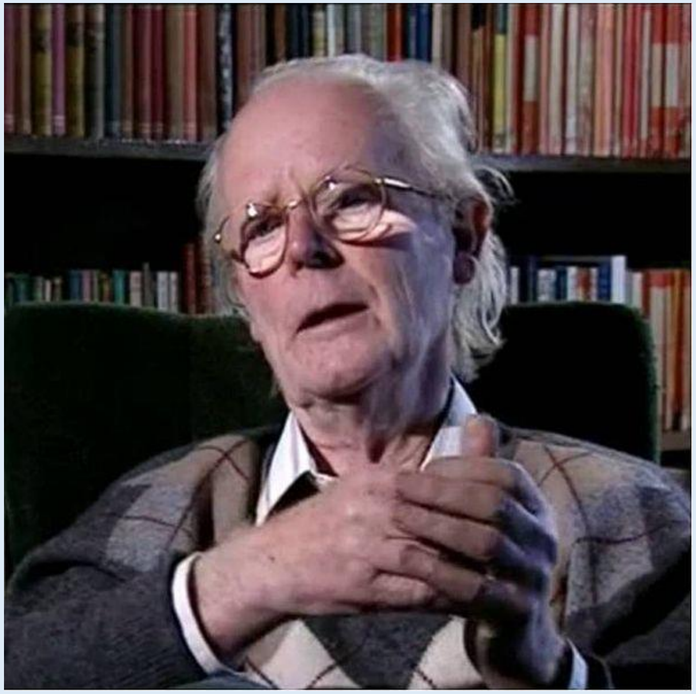
Evolution and behaviour

- Game Theory applied to evolution in the 1960s (John Maynard Smith)
- Eg in a species of bird, there might be two strategies when two birds want the same piece of food
- Fight, or “display” so that the bird which wins the display gets the food without harm to either bird



Hawks and Doves

- Some birds will always fight – “Hawks”
- Others (“Doves”) will yield to a hawk and will compete by display with another hawk
- Game theory can show the proportion of each behaviour we would expect in a population



The evolution of co-operation

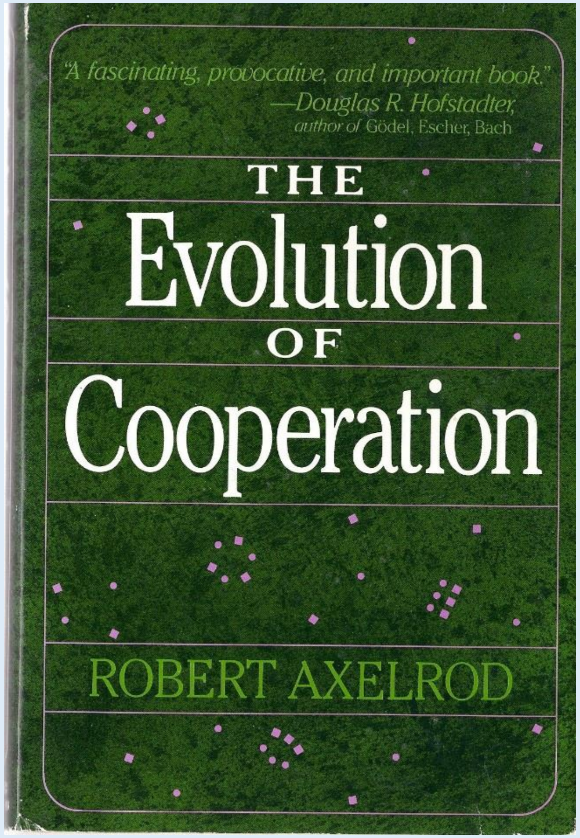
- One might think that Darwinian evolution favours selfishness (in every species)
- There is benefit in being altruistic towards one’s kin, who share one’s genes, since helping one’s family reproduce ensures more of one’s genes in future generations
- But there appears to be no evolutionary benefit in helping someone to whom one is not related

The evolution of co-operation

- And yet many species display altruism
- The problem of how altruism can thrive in a Darwinian world puzzled evolutionary theorists

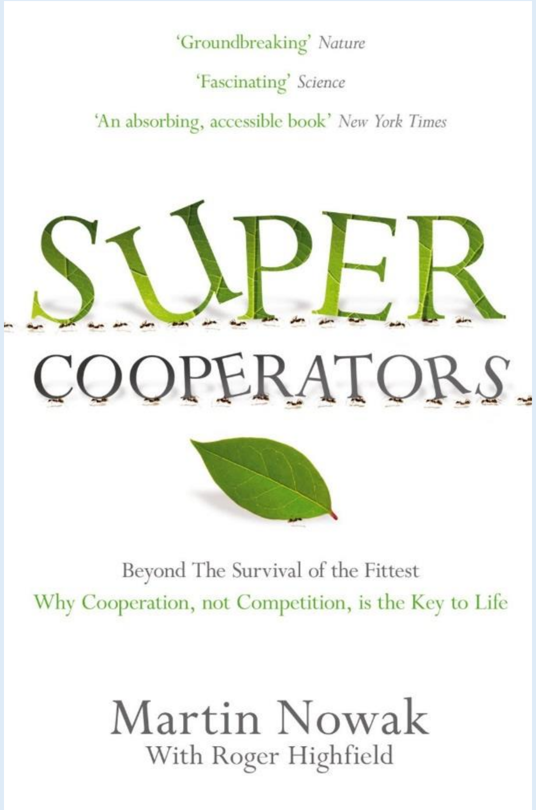
The evolution of co-operation

- Work by Robert Axelrod in the 1980s gave an answer
- Studying the Prisoner's Dilemma in game theory, he showed that "nice" strategies which don't attempt to exploit the other player do well in an evolutionary context
- We'll look at this towards the end of this course



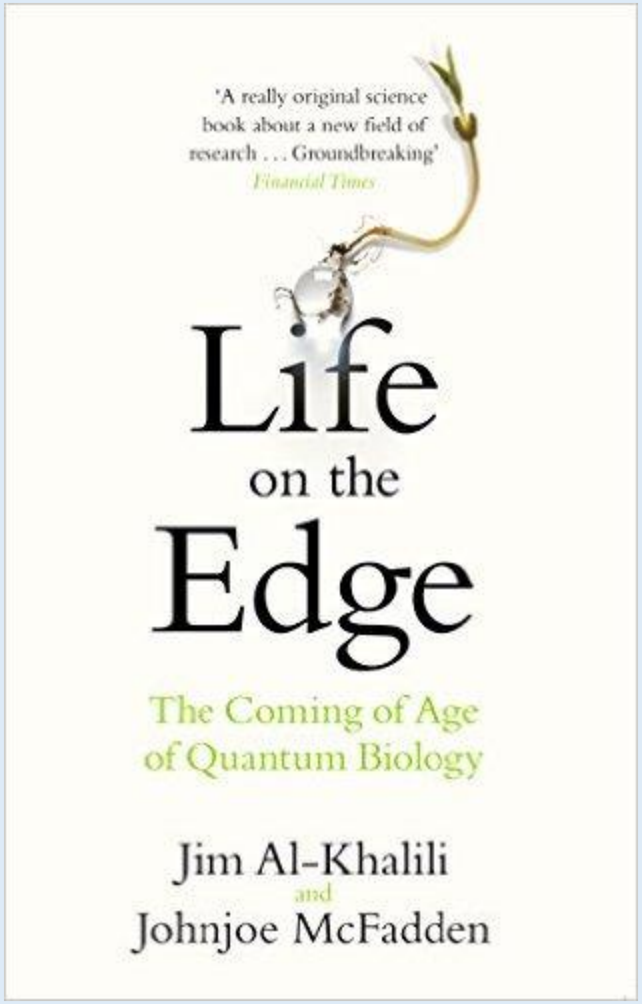
The evolution of co-operation

- In this area, game theory offered a solution to a difficult problem in evolutionary science
- And continues to offer insights into co-operation, and related issues like trust and reputation



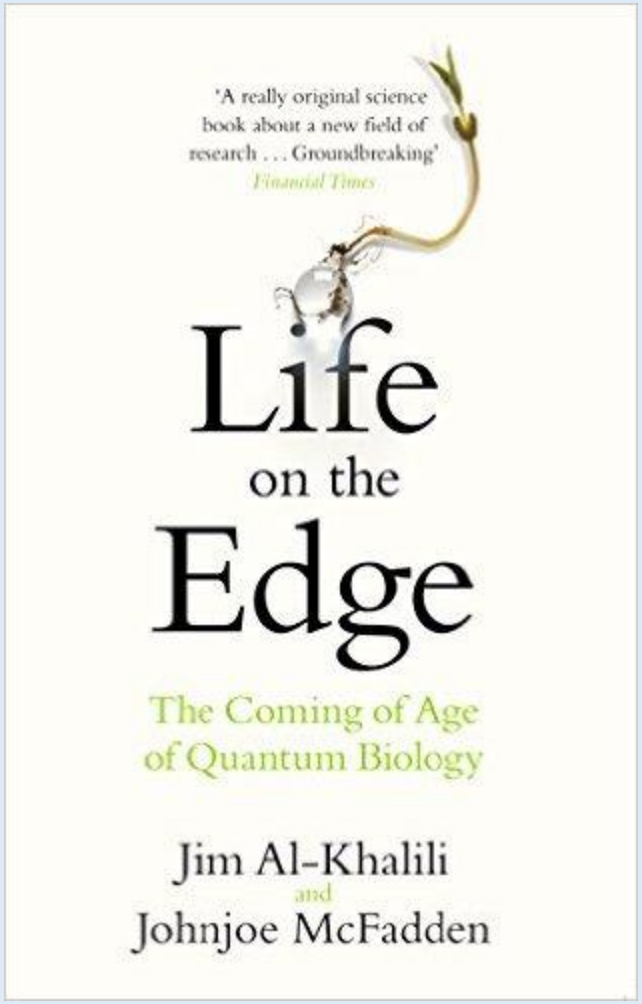
Quantum Biology

- This is perhaps the most exciting area for the future
- Quantum theory possibly explains some otherwise inexplicable biological phenomena



Quantum Biology

- How do robins navigate from Scandinavia to Africa?
- How does smell work?
- How do plants convert sunlight to energy?
- There are suggestions that at the molecular level plants are doing quantum computing!
- Al-Khalili and McFadden argue that life itself may be a quantum phenomenon, impossible in a non-quantum world



Conclusion – Mathematics in Biology

- Genetics (Mendel) – combinatorics
- Viruses – graph theory, 4D geometry, topology
- Spread of diseases – graph theory
- DNA and enzymes – knot theory
- Spots and stripes – morphogenesis, symmetry
- “Paradox of the plankton” – chaos theory
- Gaits – network theory
- Evolutionary competition, co-operation – game theory
- Robins’ navigation, smell, photosynthesis – quantum theory

Pictures

- All from Wikimedia Commons unless otherwise stated, used under the relevant Creative Commons licence
- Mendel – Hugo Iltis, 1932
- Pea plant – Rasbak
- Cowpea virus – Thomas Splettstoesser (www.scistyle.com)
- Ochiai unknot – en:User:C S (original); Pbroks13 (talk) (redraw)
- William Thurston – George M. Bergman
- Electron microscope image of DNA – Sumners, D. 1995. Lifting the curtain: Using topology to probe the hidden action of enzymes. Notices of the AMS 42:528-537.
- Haemoglobin – Zephyris
- South Chinese Tiger - J. Patrick Fischer
- Leopard – JanErkamp
- Calgary Stampede Rodeo, July 6, 2002 – JamesTeterenko
- John Maynard Smith – Web of Stories