An electro-mechanical vibration based energy harvester device is shown in the Figure here, which converts the relative motion $x(t)=u(t)-y(t)$ between the structure movement $y(t)$ and the motion of harvester mass $u(t)$ into the useful electrical energy.

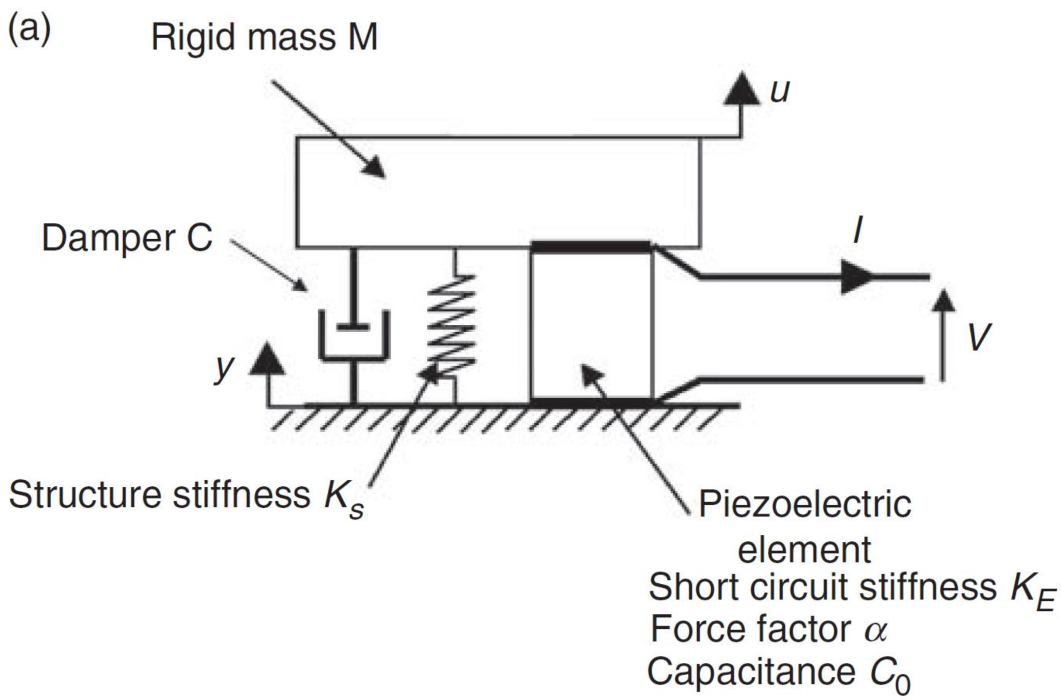

Figure: Schematic of energy harvester for mathematical modeling and design[1]

The motion of the electro-mechanical energy harvester is expressed by the following coupled differential equations of motion,

$$m\ddot{x}+c\dot{x}+kx=-m\ddot{y}-\alpha V \tag{1}$$

$$C_0\dot{V}+\frac{V}{R}=\alpha\dot{x} \tag{2}$$

where, m (kg.) is the mass of the harvester, $k=k_S+k_E$ (N/m) is its equivalent stiffness, c (N.s/m) is the equivalent damping constant and $V=IR$ (Volt) is the voltage across the resistance R (Ohm, Ω) and current I (Ampere, A) across the energy harvesting circuit. A given piezoelectric material is defined by the force factor $\alpha=-\frac{e_{33}S_0}{H}$ (N/Volt) and blocking capacitance $C_0=\frac{\varepsilon_{33}^S S_0}{H}$ (Farad, F) of the piezoelectric insert, where, e_{33}, ε_{33}^S, S_0 and H are the piezoelectric constant, permittivity, surface area and its thickness respectively.

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[1] Guyomar, D., Sebald, G. and Kuwano, H., 2011. Energy harvester of 1.5 cm^3 giving output power of 2.6 mW with only 1 G acceleration. Journal of Intelligent Material Systems and Structures, 22(5), pp.415-420.

Frequency domain analysis

(a) By taking the Laplace transform of the equations (1) and (2) obtain the following transfer functions:

(i) $\frac{X(s)}{s^2Y(s)}$ between the relative displacement and structural acceleration

(ii) $\frac{V(s)}{s^2Y(s)}$ between the output voltage and structural acceleration

Hint: $\frac{V(s)}{s^2Y(s)} = \frac{V(s)}{X(s)} . \frac{X(s)}{s^2Y(s)}$

where, $V(s), X(s)$ and $Y(s)$ is the Laplace transform of $V(t)$, $x(t)$ and $y(t)$ respectively. Assume all the initial conditions to be zero. $V(0) = x(0) = \dot{x}(0) = y(0) = \dot{y}(0) = 0$.

(b) For the given parameters: $m = 8.4\times10^{-3}$ kg, $c = 0.1541$ N.s/m, $k = 2.5\times10^{4}$ N/m, $C_0 = 18.9\times10^{-9}$ F, $R = 30000\ \Omega$, $\alpha = 1.52\times10^{-3}$ N/Volt,

Plot the absolute values transfer functions (i) $\dfrac{X(s)}{s^2Y(s)}$ and (ii) $\left|\dfrac{V(s)}{s^2Y(s)}\right|$ against the frequency, *f* (in hertz) with $s = j\omega$ where $j = \sqrt{-1}$ and ω is the frequency in rad/sec and $f = \dfrac{\omega}{2\pi}$ is frequency in Hz.

Choose the frequency range for simulation: $f = \begin{pmatrix} 0 & 0.01 & \cdots & 499.99 & 500 \end{pmatrix}$ hz.

Put two separate plots here

(c) The electro-mechanical force factor α depends type of piezoelectric materials. Suppose four different materials with: $\alpha = 1.52\times10^{-3}\begin{pmatrix}0.5 & 1 & 2 & 4\end{pmatrix}$ N/Volt, are being evaluated while keeping all the others parameters in (b) to be same. Then plot the absolute value of the transfer functions $\left|\dfrac{V(s)}{s^2Y(s)}\right|$ against the frequency, f (in hertz) for these three materials. Choose the frequency range for simulation: $f = \begin{pmatrix}0 & 0.1 & \cdots & 499.9 & 500\end{pmatrix}$ Hz.

(d) Summarize your observations of plots in (b) and (c) in relation to the design of harvester.

Time domain analysis

(e) By choosing the three state variables: $x_1 = x,\ \ x_2 = \dot{x},\ \ x_3 = V$ obtain the three state equations corresponding to equations (1) and (2). Show how you obtained these state equations and write them in the matrix form shown below.

Write down the state equations here in the following form

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{bmatrix} \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{pmatrix} \ddot{y} \qquad (3)$$

(f) Assuming the harvester system is subjected to the zero initial conditions $x(0) = \dot{x}(0) = V(0) = 0\,\text{m}$ and subjected to the structural acceleration environment $\ddot{y} = A_s \sin(\omega_s t)$ where, A_s is the acceleration magnitude of the structure and ω_s is the excitation frequency at which the structure is vibrating.

By using the state equations (3) in part (e) and values from part (c), compute the voltage $V(t)$ using Matlab ode45 function when the amplitude of the acceleration is $A_s = 9.81$ m/sec^2 and excitation frequency is 275 Hz and plot the voltage response $V(t)$ in mV (millivolt) for $t = (0 \;\; 0.001 \;\; 0.002 \;\; \cdots \;\; 0.999 \;\; 1)$ sec.

Include your properly labeled plots here.

(i) What is the maximum value of voltage output:

(ii) What is the rms (root mean square) value of voltage output:

(iii) What is the maximum harvester power output ($P = V_{max}^2 / R$) in mW (mili-watt):

(g) By repeating (f) obtain and plot the maximum harvested power output for different value of resistance in the piezoelectric circuit $R=\left(10\times10^{3}\ \ 20\times10^{3}\ \ 30\times10^{3}\ \ \cdots\ \cdots\ 10^{6}\right)\Omega$.

Hint: Use maximum harvester power output ($P=V_{\max}^{2}/R$) for each R and for better figure use "semilogx" command instead of "plot" in Matlab.

Include your properly labeled plot here.

(i) What value of resistance produces the maximum power output (in mili-Watt)

(ii) Qualitatively explain what are the parameters can influences the power output of energy harvester for a given operating condition (vibration of structure at the which harvester is mounted)

Paste your MATLAB Codes here